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Gary M. Johnson
Lewis Research Center
Cleveland, Ohio

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GARY M. JOHNSON
NASA LEWIS RESEARCH CENTER
CLEVELAND, OHIO 44135, USA

SUMMARY

A numerical procedure for the efficient simulation of steady inviscid flow is described and its utility is demonstrated. The method is uniformly valid for application in the subsonic, transonic and supersonic flow regimes. It does not rely on the introduction of additional assumptions beyond those necessary to obtain the Euler equations from the Navier-Stokes equations, nor does it make use of a time-asymptotic solution of the unsteady equations of motion.

Application of the herein-defined surrogate equation technique allows the formulation of stable, fully-conservative, type-dependent finite difference equations for use in obtaining numerical solutions to systems of first-order partial differential equations, such as the steady-state Euler equations or their various approximations.

Computational results are presented for the full Euler equations used to simulate rotational subsonic flow and for the transonic small disturbance equations. For the latter case, a computational efficiency greater than that obtained by means of the standard perturbation potential approach is indicated.

INTRODUCTION

When written in primitive variable form, the systems of partial differential equations used to describe the steady motion of an inviscid fluid are of first order and of mixed elliptic-hyperbolic type. Common examples of such systems include the transonic small disturbance equations : $\beta_x + v_y = 0 \quad , \quad v_x - u_y = 0$ (1)

where u and v are the perturbation velocity components and $\beta = (1 - M_\infty^2 - \frac{\gamma+1}{2} M_\infty^2 u)u$,

and the Euler equations : $f_x + g_y = 0$ (2)

where $f = (\rho u, \rho u^2+p, \rho uv, (E+p)u)^T \quad , \quad g = (\rho v, \rho uv, \rho v^2+p, (E+p)v)^T$,

$E = \frac{p}{\gamma-1} + \frac{\rho}{2}(u^2 + v^2)$, u and v are the velocity components, ρ is the density and p is the static pressure.

Because of the difficulties associated with both the formulation of robust finite difference analogs for such equations and the construction of stable iterative procedures for their numerical solution, these partial differential systems are not usually solved in the forms given above. Rather, as is well known, the transonic small disturbance equations are transformed into a scalar second-order partial differential equation by the introduction of a perturbation velocity potential function. The steady Euler

equations, on the other hand, are replaced by their unsteady versions, for which a temporally-asymptotic steady solution is sought, either in real or in a pseudo-time.

Relatively few departures from these approaches are to be found in the literature. Steger and Lomax (1975) have developed an iterative procedure for solving a non-conservation form of the steady Euler equations for subcritical flow with small shear. Chattot (1976) has solved the transonic small disturbance equations by differentiating them to obtain a second-order system. This work represents a special case of the aproach to be discussed here. He later adopted a variational formulation and has applied it to model problems representing the Euler equations (Chattot et al. 1979). Ozer (1977) has developed a relaxation procedure for solving the equations of motion when reformulated to yield a second-order partial differential equation in the logarithm of the pressure, together with first-order equations for the remaining variables. Blomster and Sköllermo (1977) applied Newton's method to the first-order system representing the full potential equation to solve a shockless transonic nozzle flow problem. Rizzi (1979) has extended this procedure to the steady Euler equations.

The work of these authors notwithstanding, it remains the case that contemporary numerical simulations of steady inviscid flow generally resort to either relaxation solutions of steady second-order equations in derived dependent variables or time-asymptotic solutions of unsteady first-order systems. In the former case, generality is lost, while in the latter case, the computational efficiency may be quite low.

Here we present a means by which the steady first-order, mixed-type systems of inviscid fluid flow may be readily and efficiently solved with conventional numerical techniques. The method, which we refer to as the surrogate equation technique, maintains the generality of the flow equations while allowing the use of the fully-conservative type-dependent relaxation procedures which have been developed for the efficient solution of second-order equations.

SURROGATE EQUATION TECHNIQUE

Given a first-order system, the surrogate equation technique (SET) consists of embedding this system in a second-order system (its surrogate), applying additional constraints obtained from the original system to restrict the solution set of the surrogate, and then solving the resulting partial differential problem by means of a conventional iterative procedure.

Consider a first-order system written in conservation law form, such as

$\left[\frac{\partial}{\partial x}(A\ ) + \frac{\partial}{\partial y}(B\ )\right] f = 0$ where f is an n-component vector and A and B are $n \times n$ matrices. We embed this system in a second-order surrogate of the form

$$\left[\frac{\partial}{\partial x}(M\ ) + \frac{\partial}{\partial y}(N\ )\right] \left[\frac{\partial}{\partial x}(A\ ) + \frac{\partial}{\partial y}(B\ )\right] f = 0 \quad . \tag{3}$$

This system preserves the conservation law form of the original system. Furthermore,

the jump conditions satisfied by weak solutions to the surrogate system are the same as those satisfied by the original system. The behavior of the additional characteristic directions introduced by the embedding may be controlled through the specification of the matrices M and N. For example, the choice $M = A$, $N = -B$ would result in the additional characteristics being the reflections of the original ones through the x-axis, while the choice $M = A^T$, $N = B^T$ would simply replicate the original characteristics, while symmetrizing the matrix coefficients of the terms of highest order in the surrogate second-order system. Having chosen M and N, the problem specification is completed by requiring that, in addition to satisfying the original boundary conditions of the first-order system, the solution to the surrogate system must also satisfy the first-order equations themselves at the boundaries. This is done to insure the uniqueness of the solution.

Having thus embedded the first-order system in a second-order system, we may avail ourselves of the abundance of research results on efficient, stable iterative procedures for such equations in order to construct a suitable numerical scheme. In the interest of simplicity, we confine ourselves here to the use of fully-conservative, type-dependent differencing together with the well-established successive line relaxation method. We stress, however, that SET may also be used with other iterative solution methods.

RESULTS

In the course of developing SET (Johnson 1980), we have applied it to obtain solutions to the two-dimensional steady Euler equations for purely supersonic and purely subsonic nozzle flows and for rotational subsonic flow through bends. Application has also been made to the two-dimensional transonic small disturbance equations for both subcritical and shocked supercritical flow.

Consider the Euler equations (2), rewritten in the form

$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial y}(T\;)\right] f = 0 \qquad (4)$$

where $T = BA^{-1}$ with A and B being the usual Jacobian matrices. If we choose the matrices M and N in the SET formulation (3) such that $M = I$ and $N = -T$ we obtain the second-order system shown in Fig. 1. There the application of this system to a rotational subsonic bend flow is schematically illustrated. The boundary conditions used are also shown. At the left-hand boundary, the inflow is completely specified. While this constitutes an over-specification, such treatment is considered to be adequate

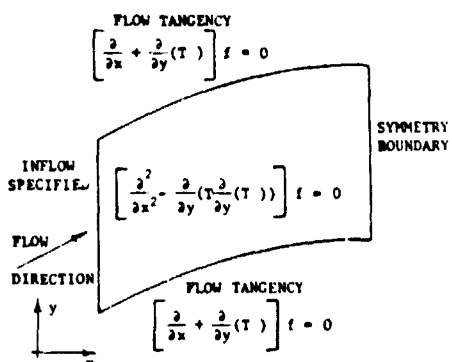

Fig. 1 SUBSONIC BEND PROBLEM

for the present, illustrative, purpose. Flow symmetry is required at the outflow boundary. On the walls, in addition to the usual flow tangency condition, satisfaction of the original first-order Euler system (4) is required. One should note that while, for simplicity, the equations presented here are written in Cartesian coordinates, the computations reported below were carried out using a slightly different form of the equations, written in sheared coordinates.

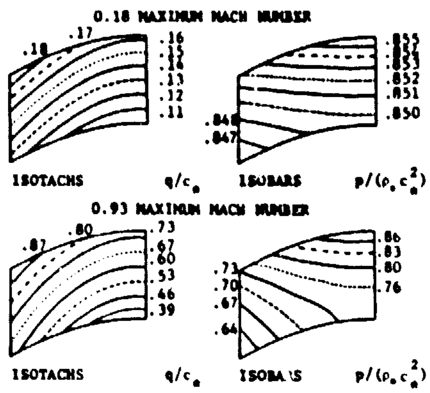

Fig. 2 SAMPLE BEND FLOW RESULTS

Shercliff (1977) presented an analytic solution to the incompressible analog of the bend flow considered here. We have solved the compressible flow problem over a range of subsonic Mach numbers. Selected results for two such Mach numbers are shown in Fig. 2. The computational domain illustrated is a section of a 90° symmetric bend, whose symmetry axis is the right-hand boundary. In its passage through the entire bend, the flow transitions between two asymptotic flows which are rectilinear shear flows. Also, as the bend cross-section reaches its maximum at the symmetry axis, the flow is decelerating in the section shown.

This case illustrates the feasability of obtaining solutions to the full Euler equations for rotational subsonic flow by means of SET. Because of the form of the matrix T used in the above formulation, it is not suitable for use in computing transonic flow. This does not appear to be an insurmountable difficulty, but, for the present, we confine our transonic flow discussion to the small disturbance equations.

Given the transonic small disturbance equations (1), we choose M and N in the SET formulation (3) such that $M = AB^{-1}$ and $N = -I$ where

$$A = \begin{bmatrix} \partial\beta/\partial u & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} .$$

This choice yields the surrogate second-order system shown in Fig. 3. This figure also illustrates the test case to which we have applied the system. We consider the flow in a two-dimensional channel with uniform inlet conditions. A circular arc airfoil surface is mounted on the lower channel wall.

The boundary conditions applied in our primary formulation of the problem (SET 1) are also shown in Fig. 3.

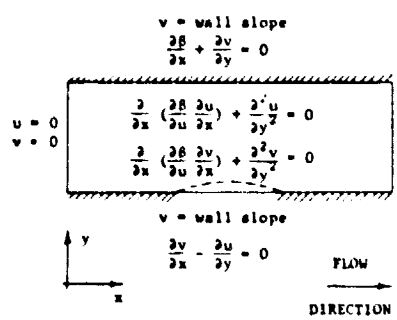

Fig. 3 TRANSONIC SMALL DISTURBANCE PROBLEM

We require both perturbation velocity components to vanish at the channel entrance and exit. The v perturbation velocity is set equal to the wall slope on both channel walls. The u perturbation velocity is obtained from the irrotationality condition on the lower wall and from the mass conservation equation on the upper wall. In an effort to probe the necessity of these boundary conditions, a secondary formulation of the problem (SET 2) was created in which the mass conservation equation on the upper wall was replaced by the irrotationality condition while all other aspects were held fixed.

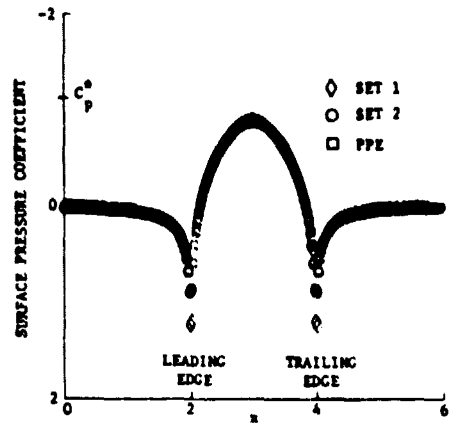

Fig. 4 SUBCRITICAL FLOW

For a subcritical flow case, the lower surface pressure coefficients resulting from both SET formulations and from the conventional Murman and Cole (1971) perturbation potential formulation (PPE) are compared in Fig. 4. All three formulations produce the same result, with SET 1 providing slightly better resolution of the stagnation points. A similar comparison for a shocked supercritical flow case is presented in Fig. 5. In this case, the agreement is also excellent, except in the immediate vicinity of the shock. Here both SET 1 and SET 2 are in agreement and both produce a very sharp shock (2 points as opposed to 4 points for PPE). However, the shock strength is underpredicted. This anomaly is presently the object of further study.

Some interesting observations may be made concerning the relative efficiencies of the three algorithms. PPE and SET 2 have roughly equivalent operations counts while that of SET 1 is approximately 1.5 times as great. An examination of convergence behavior, however, reveals a strength of the SET formulations. Fig. 6 illustrates the behavior of the maximum residual as a function of iteration level for all three formulations under the same conditions. From the data we may estimate the spectral radii for the three formulations. These, in turn, imply relative asymptotic convergence rates of 1.0, 2.3 and 8.1 for PPE, SET 2 and SET 1, respectively. Hence, we

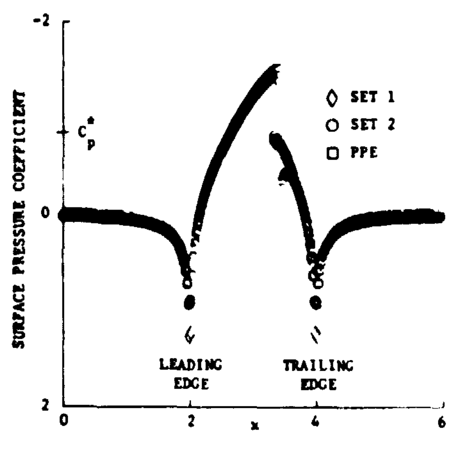

Fig. 5 SUPERCRITICAL FLOW

estimate the asymptotic computational efficiencies of PPE, SET 2 and SET 1 to be, respectively, 1.0, 2.3 and 5.4 .

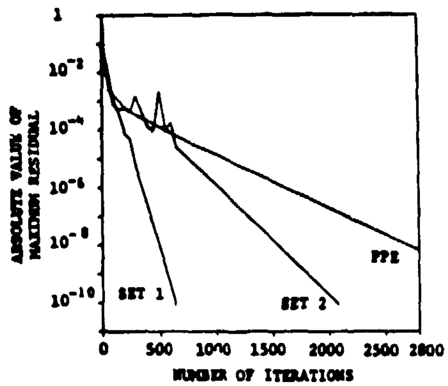

Fig. 6 CONVERGENCE BEHAVIOR

CONCLUSIONS

We have shown that it is possible to obtain a numerical solution to a system of first-order partial differential equations by solving a problem consisting of a surrogate second-order system together with the original boundary conditions and supplementary relations obtained from the first-order system.

SET provides a means for formulating problems involving first-order equations describing steady inviscid flow in such a way as to allow the use of fully-conservative type-dependent differencing and iterative solution procedures. Hence, we may solve such problems without resort either to a velocity potential or stream function or to an unsteady formulation.

An application of SET to the transonic small disturbance equations results in algorithms which, on the basis of the computational experimentation reported here, appear to have computational efficiencies which are several times greater than that of the standard perturbation potential algorithm.

In view of the successful development of iterative procedures for the solution of both the full Euler equations for subsonic and supersonic flows and the small disturbance equations for transonic flow, it appears that further applications of SET are merited.

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